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A Game Theoretic Approach to Coordination of Pricing Decisions in the Supply Chain, Including Two Types of Primary and Processed Products

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Abstract

Complementary activities such as packaging for products such as saffron significantly impact attracting more customers. This paper investigates the pricing problem in a Supply Chain (SC), including primary and processed products, in a competitive space using game theory. According to the results, the increase in complementary activities, to a certain extent, causes a decrease in profitability.

Keywords: Supply chain, Primary product, Processed product, Pricing, Game theory, Intermediary.

1 | Introduction

Consumers' access to many alternative products has intensified competition between brands, and therefore, consumers have more selection power in today's markets. On the other hand, by completing activities on the primary products such as saffron, raisins, and grains, many intermediaries can offer the processed product at a higher price to make more profit and achieve credibility among customers.

As the first important factor for customers when comparing products, choosing the optimal price can significantly impact attracting customers at first glance. Low-price selection can reduce marginal profitability, and high-price selection can cause product failure in competition. Therefore, achieving optimal pricing policies is one of the most important challenges for manufacturers and intermediaries in Supply Chains (SCs). The present paper seeks to investigate this challenge and present a solution for it under different structures. Game theory has been used to solve this problem due to its competitive logic.

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On the other hand, demand forecasting is also a problem because of the multiplicity of manufacturers and increasing product supply channels. Considering the deterministic demand in models leads to serious critiques of model logic. Therefore, in the model proposed in this study, it is necessary to assume the demand as a function of product price, alternative products, and the degree of sensitivity to the customer's price. Although the demand is considered probabilistic, the manufacturers and intermediaries have been allowed to enter their risk appetite into risk factors when ordering. These measures, in addition to considering the inter-brand competition in the chain, have brought the proposed models closer to the real-world models so that the obtained results can be applied in the real world.

Among many research studies conducted in this field, Zhao et al. [1] calculated the optimal pricing and distribution policies in a SC with two suppliers and one retailer using game theory, assuming the fuzzy parameters. Xiao and Xu [2] investigated optimal price and service level decisions in a SC with VMI policy by applying game theory. In a study, Hsieh et al. [3] explored the optimal pricing and coordination policies in a SC with a traditional retailer and several online retailers using game theory. Using game theory, Zhao et al. [4] explored optimal pricing policies for the SC with two alternative products from two manufacturers and one retailer in case of competing policies. Also, the game theory has been used by Zhao and Wang [5] to determine the optimal pricing and service policy in the SC with one supplier and two retailers in the fuzzy environment. The pricing problem in both static and dynamic states is solved with non-deterministic demand [6], [7]. Qiang et al. [8] investigated decentralized pricing in uncertain demand situations. Value-based pricing in the single-product SC, assuming random demand, including one manufacturer and one retailer, is investigated in [1], [9].

Considering price—and quality-dependent demand, Jia and Zhang [10] examined the dynamic pricing strategy in the single product and single retailer structure. Wei et al. [11] used game theory for pricing in stochastic demand situations in the two manufacturers and single retailer structure.

Wang et al. [12] investigated a closed-loop SC with one supplier and one third-party collector who compete with each other and have more pricing power than the manufacturer with production capacity constraints. The optimal pricing, recycling, and remanufacturing strategies are derived under different scenarios: The supplier-led Stackelberg game model, the third-party collector-led Stackelberg game model, and the Nash game model. Parsaeifar et al. [13] studied green SC coordination in pricing, recycling, and green product decisions using A game theory. Seyedhosseini et al. [14] proposed the Social price sensitivity of demand for a two-echelon competitive SC comprising a monopolistic manufacturer and two duopolistic retailers. In the investigated (SC), the manufacturer invests in Corporate Social Responsibility (CSR) efforts, and the retailers compete on selling price.

Assarzadegan and Rasti-Barzoki [15] introduced a pricing problem in a closed loop SC consisting of one manufacturer and two retailers in which sold items can be returned from customers in the two categories of non-defective and defective items. Malekian and Rasti-Barzokia [16] investigated price promotion and manufacturer national advertising in a manufacturer-retailer SC, taking the reference price effects of consumers into account. A centralized game is studied, followed by the two Stackelberg games of consumer price promotion and retailer-consumer price promotion. Jabarzare and Rasti-Barzoki [17] investigated a dual-channel SC comprising one manufacturer and one packaging company under price and quality-dependent demand. The manufacturer and packaging company compete on offered selling price and quality decisions. For the first time, this study investigated how the packaging company can influence the quality of products through packaging products.

Mahmoodi [18] investigated the joint decision on price and inventory control of a deteriorating product in a duopoly setting. We consider two competing SCs, each consisting of one manufacturer and one retailer. Liu et al. [19] established basic demand and profit functions by maximizing consumer utility concerning the pricing problem of two differentiated products in a dual-channel SC consisting of a dominant manufacturer and a retailer, considering different consumer network acceptance for different products. Agrawal and Yadav [20] studied an integrated production-inventory and pricing decision problem for a single manufacturer-multiple buyers SC where each buyer faces price-dependent demand. The item manufactured at a finite production rate is shipped to the buyers in multiple equal-sized shipments.

According to the literature review, no research has addressed the pricing problem during processing activities on production in a competitive environment and probabilistic demand and considering the risk factors using game theory. The following contributions can be expected from this research:

I. Considering the processed product along with the primary product

II. The possibility to increase product-added value through complementary activities by intermediary

III. To assume stochastic demand

IV. The dependency of demand on price (The dependency of processed product demand on the amount of complementary activities as well as price)

V. Considering the managers' risk-appetite factor

VI. Considering the competitive environment and using game theory to choose the optimal price and optimal level of complementary activities

2 | Model Structure

A two-level, single-period decentralized SC is proposed in the present study in which the two manufacturers produce alternative products that have the potential to add value through complementary activities. Two manufacturers supply their products at wholesale prices, and the intermediary offers the customer a new product by increasing the value added and performing a series of complementary activities in this model. On the other hand, manufacturers can directly deliver the primary product to the end customer. The present study investigates the pricing problem for the processed and the primary product.

Model assumptions

I. The primary product sales prices (Wholesale prices) to the intermediary are constant

II. Taking the demand probabilistic

III. Considering the customer loyalty level

IV. Considering the risk-appetite factor

The model structure is as *Fig. 1*.

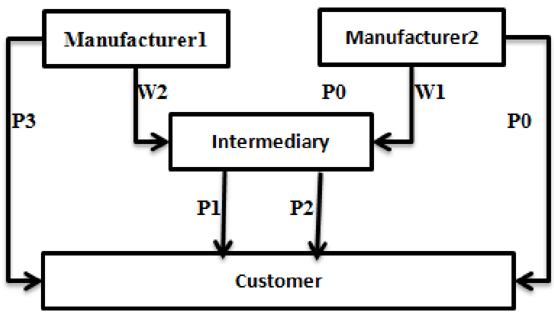

Fig. 1. The supply chain structure considered in the model.

The manufacturers 1 and 2 in *Fig. 1* supply their products at wholesale prices, W1 and W2, to the intermediary. After complementary activities, the intermediary supplies these products to the final customer at the prices P1 and P2. Manufacturers 1 and 2 have a direct sales channel for the primary product and deliver their products to the end customer through P0 and P3.

In this paper, it is tried to calculate optimal pricing policies under the following three strategies:

I. The intermediaries and manufacturers have the same decision-making power in the SC (Nash game).

II. Stackelberg game with intermediary leadership, the first scenario.

III. Stackelberg game with manufacturers leadership, the second scenario.

3 | Model Decision Parameters and Variables

The parameters and decision variables used in the model are as follows:

P_0: The primary product 1 sale price

P_1: The processed product 1 sale price

P_2: The processed product 2 sale price

P_3: The primary product 2 sale price

W_1: Wholesale price of product 1 to intermediary ($W_1 < P_0$)

W_2: Wholesale price of product 2 to intermediary ($W_2 < P_3$)

D_0: The primary product 1 demand on the internet sales channel

D_1: The processed product 1 demand

D_2: The processed product 2 demand

D_3: The primary product 2 demand

a_0: The primary product 1 demand potential

a_1: The processed product 1 demand potential

a_2: The processed product 2 demand potential

a_3: The primary product 1 demand potential

b_0: Self-sensitivity parameter to the primary product 1 price ($b_0 > 0$)

b_1: Self-sensitivity parameter to the processed product 1 price ($b_1 > 0$)

b_2: Self-sensitivity parameter to the processed product 2 price ($b_2 > 0$)

b_3: Self-sensitivity parameter to the primary product 2 price ($b_3 > 0$)

β_0: The price sensitivity parameter between the primary product 1 (Processed) demand and the processed product 1 (Primary) price ($\beta_0 > 0$)

β_1: The price sensitivity parameter between the processed product 2 demand and the primary product 1 price (And price sensitivity parameter between primary product 1 demand and the processed product 2 price) ($\beta_1 > 0$)

β_2: The price sensitivity parameter between the processed product 1 demand and the processed product 2 price and vice versa ($\beta_2 > 0$)

α_0: The price sensitivity parameter between processed product 1 demand and the primary product 1 price and vice versa ($\alpha_0 > 0$)

α_1: The price sensitivity parameter between processed product 2 demand and primary product 2 price and vice versa ($\alpha_1 > 0$)

α_2: The price sensitivity parameter between the primary product 1 price and the processed product 2 demand and vice versa ($\alpha_2 > 0$)

v_i: The amount of complementary activities performed by the intermediary to increase product value i ($i = 1,2$) ($Pi > v_i$)

$c(v_i)$: The cost function of complementary (Relationship between the amount of complementary activity for each product v_i and its associated costs) (i = 1,2)

η_i: The complementary cost factor for product i (i = 1,2)

q_0: The order quantity for the primary product 1

q_1: The order quantity for processed product 1

q_2: The order quantity for processed product 2

q_3: Order quantity for primary product 2

s_0: The value per unit of unsold inventory of the primary product 1

s_1: The value per unit of unsold inventory of the processed product 1

s_2: The value per unit of unsold inventory of the processed product 2

s_3: The value per unit of unsold inventory of the primary product 2

t_0: The shortage cost per unit of primary product 1

t_1: The shortage cost per unit of processed product 1

t_2: The shortage cost per unit of processed product 2

t_3: The shortage cost per unit of the primary product 2

m_0: The value of each unallocated unit of primary product 1 inventory at the end of the period

m_1: The value of each unallocated unit of processed product 1 inventory at the end of the period

m_2: The value of each unallocated unit of processed product 2 inventory in the intermediary channel at the end of the period

m_3: The value of each unallocated unit of processed product 2 inventory at the end of the period

πm_i (P_o, P_1, P_2, P_3): The manufacturer profit function i (i = 1,2)

$\pi m_{i(I)}$ (P_o, P_1, P_2, P_3): The manufacturer profit function from sales of primary product i (i = 1,2)

$\pi m_{i(w)}$ (P_o, P_1, P_2, P_3): The manufacturer profit function from the product i sales through wholesale to the intermediary (i = 1,2))

π_R (P_o, P_1, P_2, P_3): The intermediary profits from the sale of processed products 1 and 2

$\pi_{R(i)}$ (P_o, P_1, P_2, P_3): The intermediary profits from the processed product i (i = 1,2)

4 | Modeling

The problem is modeled according to the parameters and assumptions presented in the next stage.

4.1 | Demand Modeling

In this model, demand is first determined by a linear function of price, and the amount of complementary activity is defined. Then, it becomes stochastic on the basis of price sensitivity parameters.

Based on the defined parameters, the product 1 demand in the internet sales channel is defined as follows *Eq. (1)*:

$$D_0=a_0-b_0P_0+\beta_0(P_1-v_1)+\beta_1(P_2-v_2)+\alpha_2P_3. \quad (1)$$

Product demand 1, based on potential product demand 1 (a_o) and primary product 1 (P_o) price with impact factor b_o and processed product price 1 by the intermediary (P_1) with an impact factor of β_o, complementary activities for product 1 (v1) with an effect factor of $-\beta_o$ and the processed product 2 price by intermediary (P_2) with an impact factor of β1, complementary activities for a product of 2 by intermediary (v_2) with an impact factor of $-\beta_1$, the primary product 2 price (P_3) is defined by an impact factor of size α_2.

Processed product demand is defined by *Eq. (2)*:

$$D_1 = a_1 - b_1(P_1 - v_1) + \beta_2 P_0 + \beta_2(P_2 - v_2) + a_1 P_3. \tag{2}$$

The description of *Eq. (2)* is the same as *Eq. (1)*.

The processed product 2 is demand-defined according to *Eq. (3)*:

$$D_2 = a_2 - b_2(P_2 - v_2) + \beta_1 P_0 + \beta_2(P_1 - v_1) + a_0 P_3. \tag{3}$$

The primary product 2 demand is defined as *Eq. (4)*:

$$D_3=a_3-b_3P_3+\alpha_0(P_2-v_2)+\alpha_1(P_1-v_1)+\alpha_2P_0. \tag{4}$$

According to Hsieh and Wu [21] and Chen et al. [22], the multiplication method is used to stochastic the demand in the model. Accordingly, we use the continuous random variable x_i as a product of *Eq. (5)*, where X_i represents the probable amount of demand.

$$X_i=D_i.x_i. \tag{5}$$

4.2 | Order Quantity Modeling

The order quantity is considered one of the most important factors in determining the firm's profitability. Low-order or production quantities cause shortages, and high-order quantities cause unallocated or surplus inventory. Hence, optimizing order quantities has a significant impact on cost savings. High amounts of production or order reduce the probability of exposure to shortage but increase the probability of encountering surplus or unallocated inventory at the end of the period. The degree of risk appetite of manufacturers and intermediaries can determine the optimal production or quantity of product orders.

The different tendencies of the manufacturers and intermediaries towards the risk in the model are entered into the model through the risk parameter (z_i), according to which the order quantity of q_i is defined according to *Eq. (6)*.

$$q_i=D_i.z_i. \tag{6}$$

The low amounts of z_i indicate a decision maker with a lower risk appetite, and the lower order values and higher amounts of z_i values indicate a decision maker with a higher risk appetite and higher order values.

4.3 | The Order Deliverable Quantity Modeling

The amount a manufacturer delivers for an order quantity is not usually exactly equal to the order quantity for various reasons, such as uncertainty in demand, production and capital constraints, and competition between primary and processed products. The deliverable value for each order (q_i), Y_i, is considered a probabilistic value and is obtained by multiplying the order quantity (q_i) by a continuous random variable (y_i) according to the equation ($Y_i = q_i . y_i$).

4.4 | Profit Functions Modeling

The manufacturer i profits πm_i (P_o, P_1, P_2, P_3), for i = 1,2, can be written by *Eq. (7)* as the sum of the profits from the sale of the primary product $\pi m_{i(I)}$ (P_o, P_1, P_2, P_3) and sales of processed product $\pi m_{i(r)}$ (P_o, P_1, P_2, P_3):

$$\pi_{mi}(P_0,P_1,P_2,P_3)= \pi_{mi(I)}(P_0,P_1,P_2,P_3)+ \pi_{mi(r)}(P_0,P_1,P_2,P_3),\ (i=1,2). \quad (7)$$

The profits of the manufacturer i from the sale primary product, namely $\pi m_{i(I)}$ (P_o, P_1, P_2, P_3), are defined in *Eq. (7)* by *Eq. (8)*:

$$\pi_{mi(I)}(P_0,P_1,P_2,P_3)=E(P_j\min\{X_j,\min\{q_j,Y_j\}\}-c_j\min\{q_j,Y_j\}+m_j\max\{Y_j-q_j,0\}+s_j\max\{\min\{q_j,Yj\}-X_j,0\}-t_j\max\{X_j-\min\{q_j,Y_j\},0\}]\ ,(\text{If } i=1 \text{ then } j=0 \text{ and if } i=2 \text{ then } j=3). \quad (8)$$

The profit from the sale of the primary product i is calculated in *Eq. (8)*, by multiplying the price of the primary product i (Pj) by the amount of the primary product sold; min $\{X_j, \min\{q_j, Y_j\}\}$;, as $P_j\min\{X_j, \min\{q_j, Y_j\}\}$ considering that min $\{q_j, Y_j\}$ represents the order quantity of the primary product i, so the amount of the primary product i sales equals the minimum demand Xj and the order quantity delivered is assumed $\min\{X_j, \min\{q_j, Y_j\}\}$. The term $c_j\min\{q_j, Y_j\}$ indicates the cost of producing the primary product i based on the minimum order quantity and deliverable quantity of the primary product i. The term $m_j\max\{Y_j-q_j,0\}$ represents the proceeds from the sale of unallocated inventory of the primary product i, if any ($Y_j-q_j>0$), at the end of the period denoted by multiplying the sale price of the unallocated inventory (mj) of the primary product i by the inventory of unallocated primary product i $\max\{Y_j-q_j,0\}$. The term $s_j\max\{\min\{qj, Y_j\}-X_j,0\}$ represents the proceeds from the sale of surplus inventory at the end of the period for the primary product i. The term $t_j\max\{X_j-\min\{q_j, Y_j\},0\}]$ indicates the shortage cost of the primary product i. It is evident that a system at one point either faces a shortage $\max\{X_0-\min\{q_j, Y_j\},0\}]>0$ or a surplus $\max\{\min\{q_j, Y_j\}-X_j,\}>0$.

The profit from the sale of the primary product is defined by the manufacturer i, i.e.π_{mi}(I)(P0, P1, P2, P3) P3) according to *Eq. (9)*:

$$\pi_{mi(r)}(P_0,P_1,P_2,P_3)=E[(W_i-c_i)\min\{q_i,Y_i\}+m_i\max\{Y_i-q_i,0\}],\ i=1,2. \quad (9)$$

In *Eq. (9)* (W_i-c_i), $\min\{q_i, Y_i\}$ represents the income from the wholesale of the product i. The term $m_i\max\{Y_i-q_i,0\}]$ shows the income from unsold inventory sales of product i, if any (Yi-qi> 0) exist at the end of the period. The intermediary profit from the sale of processed product 1 and processed product 2 is calculated as *Eq. (10)*:

$$\pi_R(P_0,P_1,P_2,P_3)=\sum_{i=1}^{i=2}\pi_{R(i)}(P_0,P_1,P_2,P_3). \quad (10)$$

In *Eq. (10)*, $\pi_{R(i)}(P_o, P_1, P_2, P_3)$ is the intermediary profit from the sale of the processed product i based on *Eq. (11)*.

$$\pi_{R(i)}\ (P_0,P_1,P_2,P_3)= E(P_i\min\{X_i,\min\{q_i,Y_i\}\}-[W_i+c(v_i)]\min\{q_i,Y_i\}+s_i\max\{\min\{q_i,Y_i\}-X_i,0\}-t_i\max\{X_i-\min\{q_i,Y_i\},0\}]. \quad (11)$$

In *Eq. (11)*, the term $P_i\min\{X_i, \min\{q_i, Y_i\}\}$ is the profits from the sale of the processed product i, and $[W_i+c(v_i)]\min\{q_i, Y_i\}$ is the purchase cost at wholesale price. The cost of processing and supplementary activities for the processed product i. $s_i\max\{\min\{q_i, Y_i\}-X_i,0\}$ is the income from the sale of surplus product i by the intermediary at the end of the period and the term $t_i\max\{X_i-\min\{q_i, Y_i\},0\}]$ is the cost of the product shortage i processed by the intermediary (i = 1,2). The intermediary for completing and processing activities as much as vi for the product I need to spend c (vi). One of the most common functions to show the relationship between the level of service is presented by *Eq. (12)* [23], [24]:

$$c(v_i)=\eta_i v_i^2/2. \tag{12}$$

Using the change of variables defined by *Eqs. (13)-(16)*

$$\theta_i=E[\min\{x_i/z_i, \min\{1,y_i\}\}], \quad i= 1,2. \tag{13}$$

$$k_i=E[\max\{x_i/z_i, \min\{1,y_i\}\}], \quad i= 1,2. \tag{14}$$

$$\lambda_i=E[\min\{1,y_i\}], \quad i= 1,2. \tag{15}$$

$$k_i\geq\lambda_i\geq\theta_i>0. \tag{16}$$

The manufacturers and intermediary profit function transforms the model's *Eqs. (7)*-(10) into *Eqs. (18)* and *(19)*.

$$\pi_{m1}=[P_0\theta_0-c_0\lambda_0+m_0(1-\lambda_0)+s_0(\lambda_0-\theta_0)-t_0(k_0-\lambda_0)]z_0D_0+((w_1-c_1)\lambda_1+m_1(1-\lambda_1))z_1D_1. \tag{17}$$

$$\pi_{m2}=[P_3\theta_3-c_3\lambda_3+m_3(1-\lambda_3)+s_3(\lambda_3-\theta_3)-t3(k3-\lambda3)]z_3D_3+((w_2-c_2)\lambda_2+m_2(1-\lambda_2))z_2D_2. \tag{18}$$

$$\pi_R(P_0,P_1,P_2,P_3)=\sum_{i=1}^{i=2}\pi_{R_i}\{P_i\theta_i-[W_i+c(v_i)]\lambda_i+s_i(\lambda_i-\theta_i)-t_i(k_i-\lambda_i)\}z_iD_i. \tag{19}$$

The *Assumptions (20)-(24)* and are considered for the model parameters:

$$\pi m1, \pi m2, \pi R(1), \pi R(2) \geq 0. \tag{20}$$

$$b_0>\beta_0+\beta_1. \tag{21}$$

$$b_1>\beta_0+\beta_2+\alpha_0+\alpha. \tag{22}$$

$$b_2>\beta_1+\beta_2+\alpha_1+\alpha_2. \tag{24}$$

$$b_3>\alpha_0+\alpha_1. \tag{25}$$

The *Assumption (20)* demonstrates the need for the chain members' profitability throughout the collaboration process and to maintain its continuity.

The *Assumptions (21)-(24)* mean that the price impact of each product on its demand is greater than the sum of the price impacts of the other products.

Considering the assumptions stated above, the pricing problem can be equationted as follows *Eqs. (25) -(28)*:

$$\text{Max } \pi_{m1}, \pi_{m2}, \pi_{R1}, \pi_{R2}. \tag{25}$$

s.t.

$$P_0, P_1, P_2, P_3 \geq 0. \tag{26}$$

$$P_0\geq w_1. \tag{27}$$

$$P_3\geq w_2. \tag{28}$$

The pricing policies in the model are reviewed in the following.

5 | Model Solution

The pricing policies and model solving through game theory are reviewed in the following:

5.1 | Pricing Policy under Nash Game

The players' decision-making in the Nash game takes place simultaneously, and all players have the same decision-making power. If there is a Nash equilibrium point, players can make decisions without any ambiguity.

Definition 1 ([25]). The multiple profit function πi (x1,x2, . . . ,xn) is supermodular if and only if $\partial^2\pi_i / \partial x_i \partial x_j \geq 0$ for all x and j ≠ i. If the player's profit function is supermodular, it is called a supermodular game.

Lemma ([25]). If a supermodular game exists, there is at least one Nash equilibrium point.

Lemma 2 ([25]). If an equilibrium solution exists and the condition $\left|\partial^2\pi_k / \partial x_k \partial x_k\right| > \sum_{i=1, i\neq k}^{n} \left|\partial^2\pi_k / \partial x_k \partial x_i\right|$ is true, this point is unique.

Definition 1, *Lemma 1*, and *Lemma 2* describe the conditions for the existence of a Nash equilibrium point and its sameness in the Nash model.

Theorem 1. The profit functions π_{m1}, π_{m2}, π_R are at the supermodular in the points P_o, P_1, P_2, P_3.

By deriving the profit functions and applying the *Definition (1)*, it is proved that they are supermodular.

Theorem 2. The equilibrium point is the Nash equilibrium if *Conditions (29)-(32)* are met:

$$b_0 > (\beta_0 + \beta_1 + \alpha_2)/2. \quad (29)$$

$$b_1 > (\theta_1 z_1(\beta_2 + \beta_0 + \alpha_1) + \beta_2 z_2 \theta_2)/ 2 \theta_1 z_1. \quad (30)$$

$$b_2 > (\theta_2 z_2(\beta_2 + \beta_1 + \alpha_0) + \beta_2 z_1 \theta_1)/ 2 \theta_2 z_2. \quad (31)$$

$$b_3 > (\alpha_1 + \alpha_0 + \alpha_2)/2. \quad (32)$$

When customer sensitivity to price increases, namely when the self-sensitivity coefficients (b_o, b_1, b_2, b_3) increase and inter-product sensitivity coefficients (0, β1, β2β) decrease, the *Conditions (29)-(32)* are easily established. When the price sensitivity itself decreases, the possibility of multiple optimal solutions increases, and when the inter-product sensitivity is increased too much, the probability of a solution decreases.

Assuming the *Theorem (2)* condition is met, it is simply enough to solve the four *Eqs. (33)-(36)* to find the optimal prices in Nash game mode to reach the optimal product sale prices to the customer, namely P_o, P_1, P_2, P_3.

$$\partial\pi_R / \partial P_1 = 0. \quad (33)$$

$$\partial\pi_R / \partial P_2 = 0. \quad (34)$$

$$\partial\pi_{m1} / \partial P_0 = 0. \quad (35)$$

$$\partial\pi_{m2} / \partial P_3 = 0. \quad (36)$$

The superscript *N describes the optimal results of the model solution in the Nash game, resulting in the optimal values obtained from solving *Eqs. (33)-(36)* as (P_0^{*N}, P_1^{*N}, P_2^{*N}, P_3^{*N}).

5.2 | Pricing Policy under the Stackelberg Game

The Stackelberg game structure is used if the decision-making order and player influence differ. The player who makes the decision first and has the most power in the game is called the leader, and the second player who has the least power and decides after the leader is the follower. Accordingly, two scenarios are considered for the model in the Stackelberg game: the Stackelberg game with intermediary leadership (First scenario) and the Stackelberg game with manufacturer leadership (Second scenario).

We will discuss and explain these scenarios and determine the solution's availability and unity conditions for these two scenarios.

5.2.1 | Stackelberg game with the intermediary leadership (First scenario)

In the first scenario of the Stackelberg game, the intermediary is the leader, and the manufacturers are the follower.

Theorem 3. In the Stackelberg game with intermediary leadership, there will be a equilibrium point and it is unique if *Conditions (37)-(39)* are met according to *Lemma 2*.

$$4\theta_1 z_1\theta_2 z_2(-b_1+\frac{\beta_0{}^2}{2b_0}+\frac{2\alpha_1 b_0+\alpha_2\beta_0}{4b_0b_3-2\alpha_2{}^2}(\beta_0\frac{\alpha_2}{2b_0}+\alpha_1))\times(-b_2+\frac{\beta_1{}^2}{2b_0}+\frac{2\alpha_0 b_0+\alpha_2\beta_0}{4b_0b_3-2\alpha_2{}^2}(\beta_1\frac{\alpha_2}{2b_0}+\alpha_0))\leq (\theta_1 z_1(\beta_0\frac{\beta_1}{2b_0}+\frac{2\alpha_0 b_0+\alpha_2\beta_0}{4b_0b_3-2\alpha_2{}^2}(\beta_0\frac{\alpha_2}{2b_0}+\alpha_1)+\beta_2)+\theta_2 z_2(\beta_1\frac{\beta_0}{2b_0}+\frac{2\alpha_1 b_0+\alpha_2\beta_0}{4b_0b_3-2\alpha_2{}^2}(\beta_1\frac{\alpha_2}{2b_0}+\alpha_0)+\beta_2))^2. \quad (37)$$

$$\theta_3 z_3(-2b_3+\frac{\alpha_2^2}{2b_0})<0, \quad (38)$$

$$-2b_0 z_0 p_0\theta_0 \leq 0. \quad (39)$$

To solve the model in the Stackelberg game with the intermediary leadership, we first derive the intermediary profit *Function 19* with the derivation of P1 and P2, in which case two *Eqs. (40)* and *(41)* are obtained.

$$\partial\pi_R/\partial P_1= \theta_1 z_1(a_1-b_1(P_1-v_1)+\beta_0 P_0+\beta_2(P_2-v_2)+\alpha_1 P_3)- b_1 z_1\{P_1\theta_1-[W_1+c(v_1)]\lambda_1+s_1(\lambda_1-\theta_1)-t_1(k_1-\lambda_1)\}+ \beta_2 z_2\{P_2\theta_2-[W_2+c(v_2)]\lambda_2+s_2(\lambda_2-\theta_2)-t_2(k_2-\lambda_2)\}. \quad (40)$$

$$\partial\pi_R/\partial P_2= \beta_2 z_1\{P_1\theta_1-[W_1+c(v_1)]\lambda_1+s_1(\lambda_1-\theta_1)-t_1(k_1-\lambda_1)\}+ \theta_2 z_2(a_2-b_2(P_2-v_2)+\beta_1 P_0+\beta_2(P_1-v_1)+\alpha_0 P_3)- b_2 z_2\{P_2\theta_2-[W_2+c(v_2)]\lambda_2+s_2(\lambda_2-\theta_2)-t_2(k_2-\lambda_2)\}. \quad (41)$$

By equating zero *Eqs. (40)* and *(41)* and solving the two unknown equations in terms of P1 and P2, the optimal values of P1 and P2 are obtained.

The *S1 and * S2 superscripts represent optimal solutions in the first and second scenarios.

5.2.2 | Stackelberg game with the manufacturer leadership (Second scenario)

The manufacturer's leadership assumes the Stackelberg game structure in the second scenario, in which the players first decide on the sale price of the primary product. Finally, the intermediary determines the sale price of the processed product based on the optimal amount of the manufacturer's decision.

The condition for a single optimal solution to the problem is the given condition of the negativity of the Hessian matrix, according to which the *Theorem (4)* is obtained.

Theorem 4. In the Stackelberg game with manufacturers' leadership, there will be a equilibrium point and it is unique if *Conditions (42)-(44)* are met according to *Lemma (2):*

$$4\theta_1 z_1 b_1\theta_2 z_2 b_2 \leq (\beta_2\theta_1 z_1+\beta_2 z_2\theta_2)^2. \quad (42)$$

$$\{2\theta_0 z_0(-b_0+\beta_0{}^2/2b_1+(\theta_1 z_1(\beta_0\beta_2(z_1\theta_1+\theta_2 z_2)+2b_1\theta_2 z_2\beta_1)\}/\{(-\beta_2{}^2(\theta_1 z_1+z_2\theta_2)^2+4\theta_1 z_1 b_1\theta_2 z_2 b_2)\ \beta_0\beta_2(\theta_1 z_1+z_2\theta_2)/2\theta_1 z_1 b_1+\beta_1)\}<0. \quad (43)$$

$$2\theta_3 z_3(b_3 p_3+\alpha_0(\theta_1 z_1(\beta_2\alpha_1(z_1\theta_1+\theta_2 z_2)+2b_1\theta_2 z_2\alpha_0)/(\beta_2{}^2(\theta_1 z_1+z_2\theta_2)^2+4\theta_1 z_1 b_1\theta_2 z_2 b_2)) +\alpha_1(\theta_1 z_1(\beta_2\alpha_1(z_1\theta_1+\theta_2 z_2)+2b_1\theta_2 z_2\alpha_0)/(-\beta_2{}^2(\theta_1 z_1+z_2\theta_2)^2+4\theta_1 z_1 b_1\theta_2 z_2 b_2)+\alpha_1/2b_1)) <0. \quad (44)$$

To solve the second scenario, it is simply necessary to derive the profit function of manufacturer 1 in *Eq. (17)* of P0 and derive the profit function of manufacturer 2 in *Eq. (18)* of P3 and set it to zero based on two resulting unknown equations, P0 and P3 based on P1 and P2. Put P0 and P3 in the intermediary profit function of *Eq. (18)* and derive this function once for P1 and once for P2. We solve the two unknown equations based on P1 and P2. The optimal values of the retail prices P1 and P2 are also calculated by substituting the optimal values of P0 and P3, and the optimal pricing policy is obtained.

After calculating the optimum values of prices based on different strategies and maximizing profit, the demand values are extracted in optimum conditions by placing the obtained values in demand functions. The optimal order quantities and deliverables can be obtained based on the calculated demand values considering the risk factor.

5.3 | Numerical Example

It is assumed in the numerical example that two manufacturers produce quite similar and, at the same time, different products.

Theorems (2)-(4) are checked when selecting the problem parameters to ensure that there is a balanced equilibrium point in the Nash and Stackelberg games.

The profit functions will be according to *Eqs. (45)-(47)*, assuming the products are the same.

$$\pi_R(p_0,p_1,p_2,p_3)= \{P_1\theta-[W+c(v)]\lambda+s(\lambda-\theta)-t(k-\lambda)\}z \quad (a-b(P_1-v)+\beta P_0+\beta(P_2-v)+\beta P_3) +\{P_2\theta-[W+c(v)]\lambda+s(\lambda-\theta)-t(k-\lambda)\}z(a-b(P_2-v)+\beta P_0+\beta(P_1-v)+\beta P_3). \quad (45)$$

$$\pi_{m1}=[P_0\theta-c\lambda+m(1-\lambda)+s(\lambda-\theta)-t(k-\lambda)]z(a-bP_0+\beta(P_1-v)+\beta(P_2-v)+\beta P_3)+((w-c)\lambda+m(1-\lambda))z(a-b(P_1-v)+\beta P_0+\beta(P_2-v)+\beta P_3). \quad (46)$$

$$\pi_{m2}=[P_3\theta-c\lambda+m(1-\lambda)+s(\lambda-\theta)-t(k-\lambda)]z(a-bP_3+\beta(P_2-v)+\beta(P_1-v)+\beta P_0)+((w-c)\lambda+m(1-\lambda))z(a-b(P_2-v)+\beta P_0+\beta(P_1-v)+\beta P_3). \quad (47)$$

The numerical values of the model numerical example parameters are the development of the numerical example of Kurata et al. [26], Hsieh and Wu [21], and Chen et al. [22], which are considered as follows.

$a=100,\ b=10,\ \beta=4,\ \alpha=4,\ m=0.5,\ s=0.5,\ t=0.5,\ c=6,\ w=9,\ v=3,\ z=1,\ \eta=0.5,$
$cv=\eta v^2/2=2.25.$

As mentioned above, to probablize the demand and the deliverable rate for orders, we will use the continuous random variables x and y with an average of 1. The coefficient of variation for these random variables is as follows:

$CVx=0.35, \quad CVy=0.2.$

It is assumed that these variables have a uniform random distribution over the interval, $[1-\tilde{x}, 1+\tilde{x}]$and $[1-\tilde{y}, 1+\tilde{y}]$.

According to the numbers and data obtained, the values of the change of the problem variables are calculated as follows:

$\vartheta=E[\min\{x_i/z, \min\{1,y_i\}\}] = 0.81.$

$k=E[\max\{xi/z, \min \{1,yi\}\}]= 1.11.$

$\lambda=E[\min\{1,yi\}]= 0.92.$

By using the Nash game, the following four equations are derived by placing parameters and example variables in the derivatives of the profit functions.

By solving the above equations, the following solutions for optimal prices are obtained in the Nash game:

$$\begin{cases} 3.24P_0-16.2P_1+6.48P_2+3.24P_3=-127.96, \\ 3.24P_0+6.48P_1-16.2P_2+3.24P_3=-127.96, \\ -16.2P_0+3.24P_1+3.24P_2+3.24P_3=-133.08, \\ 3.24P_0-16.2P_1+3.24P_2-16.2P_3=-133.08, \end{cases}$$

According to the prices obtained, the profits are as follows:

$\pi_{m1}{}^{N*}$=2878.625.

$\pi_{m2}{}^{N*}$=2878.625.

$\pi_R{}^{N*}$=5006.168.

As the products were considered to be exactly the same, as expected, identical prices were obtained for the same products and the manufacturers' profits were also equal.

Table 1 indicates the optimal values of the profit functions of solving problems in different game structures.

Table 1. The results of the numerical example solution.

Scenario	π_{m1}*	π_{m2}*	π_r*
Nash game	2878	2878	5006
First scenario	7479	7479	5564
Second Scenario	4325	4325	5904

According to *Table 1*, the manufacturers and intermediaries make the most profits in this example when they enter the pricing game as followers of the Stackelberg game. Contrary to popular belief, having more market power and being a leader in the game does not always result in more profit.

To investigate the effect of sensitivity parameters changes to price on profit and prices, each of the price sensitivity parameters has been changed to ± 0.30, and the results are shown in *Fig. 1*.

Fig.1 illustrates the impact of parameter variations on the amount of complementary activities on Intermediary profit under different scenarios.

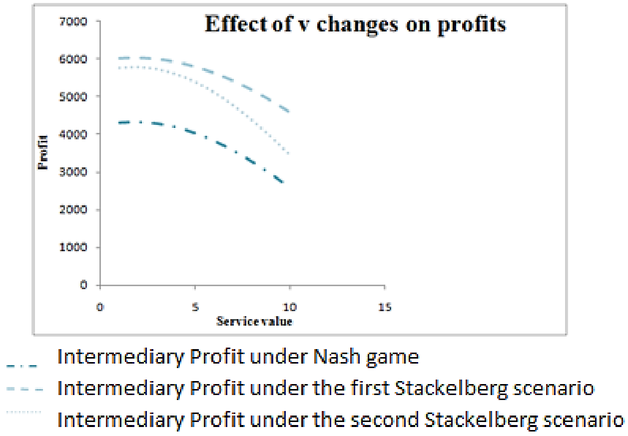

Fig. 1. The impact of parameter variations on the amount of complementary activities on Intermediary profit under different scenarios.

According to *Fig. 1*, an increase in the number of complementary activities can partially increase profitability, but its excessive increase reduces the intermediary profit due to its impact on the processed product price because the increased prices can reduce the desirability of products to customers. On the other hand, a decrease in the complementary activities level due to a reduction in the price of the finished product can increase the product's desirability for the customer and the intermediary's profitability. The optimal amount can be obtained by solving the profit equations in the game based on the optimal level of complementary activities.

6 | Conclusion

The problem of pricing and inventory in a decentralized SC with two manufacturers and one intermediary in a single-period, two- level structure with stochastic demand and price dependence in which two different brands for the primary product (1 and 2) and the processed product is investigated in the present paper. It was assumed that the primary product would be delivered to the final customer through the manufacturers and the processed products through the intermediary after complementary activities. The amount of complementary activities carried out affects the price, and the price affects the product demand. There is competition between intermediaries and manufacturers. Three types of power structures were used for the competitive environment based on the conditions governing the game theory problem. The solution model was determined using three kinds of Nash and Stackelberg game strategies with two different leadership types (Intermediary leadership and manufacturer leadership), and the conditions for solutions availability and uniqueness were met. For further analysis, the model was solved for the case of completely similar products, and the optimal results and conditions were extracted. According to the research results, The problem can be addressed by having a unique answer by increasing the price self-sensitivity parameter in the customer and reducing the price sensitivity parameters between products. Still, it is possible to increase the price sensitivity parameters too much and to reduce the price sensitivity parameters too much, providing the conditions for the problem to be optimized without a solution.

On the other hand, multiple optimal solutions are possible by decreasing the self-sensitivity parameter and increasing the price sensitivity parameters between products. One way to reduce the price sensitivity of the intermediary channel is to provide the customer with a better-processed product or, in other words, to increase the number of complementary activities performed on the product. The threshold for increasing the amount of complementary activity is easily visible in the numerical example, which means that, despite the positive effect of increasing the level of complementary activity in reducing self-sensitivity to price, its excessive increase causes the price increase and the customer's dissatisfaction with the product due to the high price.

7 | Suggestions

Considering the model assumptions, the following suggestions can be proposed for future research:

I. Considering several intermediaries for the chain and the possibility of an Internet channel for them

II. Considering fuzzy price sensitivity parameters

III. Assuming multi-period models rather than single-period

IV. Considering the complementary activities level as the decision variable to determine the optimal service level

V. Considering the wholesale price as a variable

References

[1] Zhao, J., Liu, W., & Wei, J. (2013). Competition under manufacturer service and price in fuzzy environments. *Knowledge-based systems*, *50*, 121–133. https://doi.org/10.1016/j.knosys.2013.06.003

[2] Xiao, T., & Xu, T. (2013). Coordinating price and service level decisions for a supply chain with deteriorating item under vendor managed inventory. *International journal of production economics*, *145*(2), 743–752. https://doi.org/10.1016/j.ijpe.2013.06.004

[3] Hsieh, C. C., Chang, Y. L., & Wu, C. H. (2014). Competitive pricing and ordering decisions in a multiple-channel supply chain. *International journal of production economics*, *154*, 156–165. https://doi.org/10.1016/j.ijpe.2014.04.024

[4] Zhao, J., Wei, J., & Li, Y. (2014). Pricing decisions for substitutable products in a two-echelon supply chain with firms' different channel powers. *International journal of production economics*, *153*, 243–252. https://doi.org/10.1016/j.ijpe.2014.03.005

[5] Zhao, J., & Wang, L. (2015). Pricing and retail service decisions in fuzzy uncertainty environments. *Applied mathematics and computation*, *250*, 580–592. https://doi.org/10.1016/j.amc.2014.11.005

[6] Zhang, J., Nault, B. R., Yang, W., & Tu, Y. (2012). Dynamic price quotation in a responsive supply chain for one-of-a-kind production. *International journal of production economics*, *139*(1), 275–287. https://doi.org/10.1016/j.ijpe.2012.05.011

[7] Şen, A. (2013). A comparison of fixed and dynamic pricing policies in revenue management. *Omega*, *41*(3), 586–597. https://doi.org/10.1016/j.omega.2012.08.001

[8] Qiang, Q., Ke, K., Anderson, T., & Dong, J. (2013). The closed-loop supply chain network with competition, distribution channel investment, and uncertainties. *Omega*, *41*(2), 186–194. https://doi.org/10.1016/j.omega.2011.08.011

[9] Zhao, Y., Yang, L., Cheng, T. C. E., Ma, L., & Shao, X. (2013). A value-based approach to option pricing: The case of supply chain options. *International journal of production economics*, *143*(1), 171–177. https://doi.org/10.1016/j.ijpe.2013.01.004

[10] Jia, J., & Zhang, J. (2013). Dynamic ordering and pricing strategies in a two-tier multi-generation durable goods supply chain. *International journal of production economics*, *144*(1), 135–142. https://doi.org/10.1016/j.ijpe.2013.01.029

[11] Wei, Y., Hu, Q., & Xu, C. (2013). Ordering, pricing and allocation in a service supply chain. *International journal of production economics*, *144*(2), 590–598. https://doi.org/10.1016/j.ijpe.2013.04.022

[12] Wang, J., Zhou, Z., & Yu, M. (2019). Pricing models in a sustainable supply chain with capacity constraint. *Journal of cleaner production*, *222*, 57–76. https://doi.org/10.1016/j.jclepro.2019.01.319

[13] Parsaeifar, S., Bozorgi-Amiri, A., Naimi-Sadigh, A., & Sangari, M. S. (2019). A game theoretical for coordination of pricing, recycling, and green product decisions in the supply chain. *Journal of cleaner production*, *226*, 37–49. https://doi.org/10.1016/j.jclepro.2019.03.343

[14] Seyedhosseini, S. M., Hosseini-Motlagh, S.-M., Johari, M., & Jazinaninejad, M. (2019). Social price-sensitivity of demand for competitive supply chain coordination. *Computers \& industrial engineering*, *135*, 1103–1126. https://doi.org/10.1016/j.cie.2019.05.019

[15] Assarzadegan, P., & Rasti-Barzoki, M. (2020). A game theoretic approach for pricing under a return policy and a money back guarantee in a closed loop supply chain. *International journal of production economics*, *222*, 107486. https://doi.org/10.1016/j.ijpe.2019.09.007

[16] Malekian, Y., & Rasti-Barzoki, M. (2019). A game theoretic approach to coordinate price promotion and advertising policies with reference price effects in a two-echelon supply chain. *Journal of retailing and consumer services*, *51*, 114–128. https://doi.org/10.1016/j.jretconser.2019.05.028

[17] Jabarzare, N., & Rasti-Barzoki, M. (2020). A game theoretic approach for pricing and determining quality level through coordination contracts in a dual-channel supply chain including manufacturer and packaging company. *International journal of production economics*, *221*, 107480. https://doi.org/10.1016/j.ijpe.2019.09.001

[18] Mahmoodi, A. (2020). Stackelberg-Nash equilibrium of pricing and inventory decisions in duopoly supply chains using a nested evolutionary algorithm. *Applied soft computing*, *86*, 105922. https://doi.org/10.1016/j.asoc.2019.105922

[19] Liu, Y., Li, J., Ren, W., & Forrest, J. Y.-L. (2020). Differentiated products pricing with consumer network acceptance in a dual-channel supply chain. *Electronic commerce research and applications*, *39*, 100915. https://doi.org/10.1016/j.elerap.2019.100915

[20] Agrawal, A. K., & Yadav, S. (2020). Price and profit structuring for single manufacturer multi-buyer integrated inventory supply chain under price-sensitive demand condition. *Computers & industrial engineering*, *139*, 106208. https://doi.org/10.1016/j.cie.2019.106208

[21] Hsieh, C. C., & Wu, C. H. (2009). Coordinated decisions for substitutable products in a common retailer supply chain. *European journal of operational research*, *196*(1), 273–288. https://doi.org/10.1016/j.ejor.2008.02.019

[22] Chen, Y. C., Fang, S. C., & Wen, U. P. (2013). Pricing policies for substitutable products in a supply chain with Internet and traditional channels. *European journal of operational research*, *224*(3), 542–551. https://doi.org/10.1016/j.ejor.2012.09.003

[23] Yao, D. Q., & Liu, J. J. (2005). Competitive pricing of mixed retail and e-tail distribution channels. *Omega*, *33*(3), 235–247. https://doi.org/10.1016/j.omega.2004.04.007

[24] Yao, D. Q., Yue, X., & Liu, J. (2008). Vertical cost information sharing in a supply chain with value-adding retailers. *Omega*, *36*(5), 838–851. https://doi.org/10.1016/j.omega.2006.04.003

[25] Cachon, G. P., & Netessine, S. (2006). Game theory in supply chain analysis. *Models, methods, and applications for innovative decision making*, 200–233. https://doi.org/10.1287/educ.1063.0023

[26] Kurata, H., Yao, D. Q., & Liu, J. J. (2007). Pricing policies under direct vs. Indirect channel competition and national vs. store brand competition. *European journal of operational research*, *180*(1), 262–281. https://doi.org/10.1016/j.ejor.2006.04.002